

Supplying Dark Energy from Scalar Field Dark Matter

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Abstract

We consider the hypothesis that the dark matter consists of ultra-light bosons residing in the state of a Bose-Einstein condensate, which behaves as a single coherent wave rather than as individual particles. In galaxies, spatial distribution of scalar field dark matter can be described by the relativistic Klein-Gordon equation on a background space-time with Schwarzschild metric. In such a setup, the equation of state of scalar field dark matter is found to be changing along with galactocentric distance from dust-like ($p = 0$), inside the galactic halo, to vacuum-like ($p = -\rho$), in intergalactic space. We reveal the ranges of masses and self-interaction strengths of scalar field that allow the Bose-Einstein condensate to supply both dark matter and dark energy components of the universe.

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Although, the Λ CDM model explains very successfully numerous observations on both the large scale structure and galaxies formation, the microscopic nature of the dark matter (DM) is still unknown. In the most common paradigm, DM constituents are so called weakly interacting massive particles, while attempts to detect them directly or indirectly yet allowed only to obtain restrictions on the basis of constraints coming from non-observations of signals over backgrounds of different instruments. Moreover, it turns out that although the standard Λ CDM can successfully accommodate the large scale structure formation through the evolution of DM fluctuations, there are still some poorly understood issues on galactic and sub-galactic scales, such as cuspy halos [1, 2], missing satellite problem [3] and too big to fail problems [4-6].

All these small-scale structure anomalies can in principle be resolved if the DM is made up of ultralight bosons that form a Bose-Einstein condensate (BEC), i.e. a single coherent macroscopic wave function with long range correlation [7–17]. Such kinds of bosons are usually modeled by a scalar field either with or without self-interaction. In case the scalar field DM (SFDM) does not possess any self-interaction, the quantum pressure of localized particles would be sufficient to stabilize the DM halo against gravitational collapse only for very light particles with mass $m \sim 10^{-22}\text{eV}$ [18–22]. From the other side, introduction of a small repulsive self-interaction makes possible to extend the range of allowed masses of the SFDM particles up to $m \leq 1\text{ eV}$ [23–27]. Possible particle physics examples for bosons constituting SFDM are weakly interacting slim particles [28], which include axions and axion-like particles [29–36] and spin-1 hidden bosons from string theory [37, 38]. Below we do not refer to any specific particle physics model, making our subsequent discussion generically applicable to any SFDM with repulsive self-interaction¹.

In this paper the distribution of BEC of ultra-light scalar field in outer galactic regions is explored. We argue that SFDM, together with its success in resolving of the small scale structure ΛCDM anomalies, mentioned above, can supply the vacuum like equation state inherent to the dark energy governing the contemporary expansion of the universe.

We assume that the universe is filled by a nonlinear complex scalar field Φ with the Lagrangian,

$$L_\Phi = \frac{1}{2}g^{\mu\nu}D_\mu\Phi^*D_\nu\Phi - V(\Phi^*\Phi) , \quad (1)$$

where D_μ denotes covariant derivatives, we use the system of units where $c = \hbar = 8\pi G = 1$ and self-interacting potential has the form:

$$V(\Phi^*\Phi) = \frac{m^2}{2}\Phi^*\Phi + \frac{\lambda}{4}(\Phi^*\Phi)^2 . \quad (2)$$

The energy momentum tensor of scalar field associate with the Lagrangian (1) has the form:

$$T_{\mu\nu} = \text{diag}(\rho, p, p, p) = D_\mu\Phi^*D_\nu\Phi + D_\nu\Phi D_\mu\Phi^* - 2g_{\mu\nu}L_\Phi , \quad (3)$$

where the energy density and pressure are given by:

$$\begin{aligned} \rho &= T_0^0 = D_t\Phi^*D_t\Phi + V , \\ p &= T_i^i = -D_t\Phi^*D_t\Phi + V . \end{aligned} \quad (4)$$

For the Lagrangian (1) one can write down the Klein-Gordon equation,

$$\left[\frac{1}{\sqrt{-g}}\partial_\mu (\sqrt{-g}g^{\mu\nu}\partial_\nu) + m^2 + \lambda N \right] \Phi = 0 , \quad (5)$$

where it was introduced the dimensional particle number function,

$$N = \Phi^*\Phi . \quad (6)$$

¹Actually, to ensure the radiative stability of the ultra-light scalar its potential should possess an approximate symmetry which usually gives rise to an attractive self-interaction. However, it is also possible to have a realistic model of an ultra-light scalar with repulsive self-interaction [27, 39].

We study scalar field, gravitationally trapped in galaxies, using the Schwarzschild metric,

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) , \quad (7)$$

where

$$f(r) = 1 - \frac{2D}{r} . \quad (8)$$

Here, the parameter D , in our units, has the dimension of inverse mass. For a typical galaxy one obtains:

$$D \sim 10^{12} \frac{M_\odot}{M_{Pl}^2} \sim 10^{33} \text{ GeV}^{-1} , \quad (9)$$

where M_{Pl} and M_\odot are the Planck and solar masses, respectively.

The Schwarzschild space-time (7) is highly symmetric, so in the Klein-Gordon equation (5) we can factorize the variables as

$$\Phi = Ae^{i\omega t} Y_{lm}(\theta, \phi) \psi(r) , \quad (10)$$

where A is the normalization constant and $Y_{lm}(\theta, \phi)$ denotes spherical harmonics. For the wave function (10) we assumed harmonic time dependence, which is valid for current studies. However, to investigate the cosmological evolution of the system, one would need to use a general, time dependent, scalar field.

Let us restrict our attention to the monopolar component of the wave function and consider the scalar field with zero angular momentum, $l = 0$, so that (10) can be normalized as

$$\Phi = e^{i\omega t} \psi(r) . \quad (11)$$

In this case the Klein-Gordon equation (5) for the radial function $\psi(r)$ takes the form:

$$\left[f \partial_r^2 + \frac{1+f}{r} \partial_r + \frac{\omega^2}{f} - \frac{1-f}{r^2} - m^2 - \lambda N \right] \psi(r) = 0 . \quad (12)$$

Another simple transformation of the wave function,

$$\psi(r) \equiv \frac{u(r)}{r} , \quad (13)$$

and the introduction of the Regge-Wheeler tortoise coordinate,

$$R = \int \frac{dr}{f} = r + 2M \ln \left(\frac{r}{2M} - 1 \right) , \quad (14)$$

brings (12) to a non-linear Schrödinger-like equation, which is a kind of Gross-Pitaevskii equation [12–15]:

$$-u'' + (V_{eff} + \lambda f N) u = \omega^2 u , \quad (15)$$

where primes denote derivatives with respect to the new radial coordinate R and

$$V_{eff} = f \left(m^2 + \frac{f'}{R} \right) . \quad (16)$$

In general, to investigate the physical properties of BEC usually the Gross-Pitaevskii equation is used. However, for a scalar field, under some conditions, as one sees from (15), BEC can be obtained also from the Klein-Gordon equation [16, 17, 40–42]. This kind of non-linear equations are known for having soliton-like solutions corresponding to BEC.

A formal solution of Gross-Pitaevskii equations for the quasi-stationary field distributions can be obtained within so called Thomas-Fermi approximation (by neglecting kinetic energy terms), which is very useful for exploring properties of BEC and is valid for sufficiently large condensate clouds if the system is diluted enough. In Thomas-Fermi approximation,

$$(V_{eff} + \lambda f N) u = \omega^2 u , \quad (17)$$

i.e. when $u'' \rightarrow 0$ in the Gross-Pitaevskii-like equation (15), we obtain the following solution for the particle number density function (6):

$$N = \frac{u^2}{r^2} = \frac{\omega^2 - V_{eff}}{\lambda f} . \quad (18)$$

Therefore, the size of the scalar field condensate, d , which we want to interpret as a galactic DM halo, can be calculated from the condition:

$$N = 0 . \quad (19)$$

For sufficiently large clouds,

$$d \gg D , \quad R \sim r , \quad (20)$$

from (16), (18) and (19) we find:

$$d \sim \frac{M^{1/3}}{\omega^{2/3}} . \quad (21)$$

So the size of the galactic DM halo is described by the mass of the galaxy (9) and by the kinetic term in the scalar field Lagrangian (1) (which is proportional to ω^2). To obtain the realistic size of the BEC DM clouds within galaxies, namely

$$d \sim 30 \text{ kpc} , \quad (22)$$

from (21), the energy (mass) of the ultra-light scalar particles should be taken around

$$\omega \sim m \sim 10^{-38} \text{ GeV} . \quad (23)$$

Note that this value of the mass is lower than that typically considered in non-selfinteracting SFDM models [18–22]. This difference is coming from the fact that the wavelength of the BAC constituents is taken as the distance of maximal dilution of the condensate measured from the center of the Schwarzschild space-time rather than as the de Broglie wavelength typical for a galactic halo as accepted in [18–22].

For the parametrization of the galactic DM halo energy density distribution, ρ , the NFW profile is commonly used [43, 44]. While the parameters and shapes of ρ in different models may differ from the NFW profile [45–48], the asymptotic $1/r^3$ behavior in outer galactic regions, in the range of galactocentric distances $r \simeq 10 - 100 \text{ kpc}$, is widely accepted. Using asymptotic value of our effective potential (16), which for the scalar field number density function (18) gives

$$N_{(r \gg M)} \sim \frac{1}{\lambda} \left(\omega^2 - \frac{2M}{r^3} \right) , \quad (24)$$

and the expression for the SFDM energy density as

$$\rho = T_0^0 = \frac{f}{4} \frac{N'^2}{N} + \left(\frac{\omega^2}{f} + m^2 \right) N + \frac{\lambda}{2} N^2 , \quad (25)$$

which follows from the definition of scalar field energy-momentum tensor (3) in the Schwarzschild background space-time (7), one can see that the SFDM density (25) distribution, for large distances (18), indeed obeys the NFW behavior.

Now, let us consider the large distance tail of the SFDM condensate (25) in outer galactic region $r \geq 100$ kpc, where the density of DM particle is very low,

$$N^2 \rightarrow 0 , \quad (26)$$

while not exactly zero. In this case, we can neglect the nonlinear term in the Klein-Gordon equation (12) and also assume that momentums of scalar particles, together with the angular momentums, are zero, so we can use the dispersion relation:

$$\omega^2 = m^2 . \quad (27)$$

Thus, the radial Klein-Gordon equation (12) takes the simplest form,

$$\left(\partial_r^2 + \frac{2}{r} \partial_r \right) \psi(r) = 0 , \quad (28)$$

with the constant solution,

$$\psi_\infty = C , \quad (29)$$

where C is the integration constant. So, in intergalactic space, we stay with the oscillatory wave function (11),

$$\Phi_\infty \sim C e^{\pm i m t} , \quad (30)$$

which brings the scalar field Lagrangian (1) to the following form:

$$L_\infty = -\frac{1}{4} \lambda C^4 . \quad (31)$$

Therefore, non-zero components of the scalar field energy-momentum tensor (4) can be expressed as

$$\begin{aligned} T_0^0 &= \rho = 2m^2 C^2 - L_\infty , \\ T_i^i &= -p = -L_\infty . \end{aligned} \quad (32)$$

In other words, while the size of the galactic DM (21) is described by the kinetic term of order of ω^2 , the outer galactic regions are dominated by the non-linear part of the DM scalar field potential $\sim L_\infty$. Assuming that

$$\lambda C^2 \gg 8m^2 , \quad (33)$$

one can neglect the first term of T_0^0 in (32), which implies vacuum-like equation of state:

$$\rho = -p = -L_\infty . \quad (34)$$

From the expression

$$\rho + 3p = 2C^2 \left(m^2 - \frac{1}{4}\lambda C^2 \right) \quad (35)$$

one can see that for the case (33) the strong energy condition is broken and the universe is in the acceleration regime. Thus, one can identify the asymptotic value of SFDM potential (31) with the cosmological constant Λ ,

$$\frac{1}{4}\lambda C^4 \equiv \Lambda \sim 10^{-48} \text{ GeV}^4, \quad (36)$$

which is commonly used as a dark energy candidate.

To estimate the values of the integration and coupling constants, C and λ , note that in outer galactic region the scalar particle number function should be very small (26), $N = C^2 \ll 1$. Thus, from (36) we find,

$$10^{-23} \text{ GeV}^2 < C^2 < 0.1 \text{ GeV}^2, \quad 10^{-46} < \lambda < 0.1, \quad (37)$$

provided that $\lambda \ll 1$. These values also are compatible with the assumption (33).

We emphasize that the estimation (37) for the modulus of the SFDM in the outer galactic region (30) is valid only for the Schwarzschild space-time and oscillatory time depended wave function (10). A similar requirements of the time evolution of the BEC wave function has been applied in [13], while the self-interaction imposed on the constituent ultra-light scalar boson was taken as attractive one. The value of the mass of the ultra light bosons needed for realization of our scenario is of the same order of magnitude used in model [14], which also suggest that a macroscopic wavefunction of a BEC gives rise to a vacuum-like state which is typical for a positive cosmological constant. To explore cosmological features of our model one should use the Robertson-Walker metric and evolving in time scalar fields. Cosmological studies of the BEC DM show significant differences with respect to the standard cosmology. Presence of such condensates could have modified drastically the cosmological evolution of the early universe, as well as the large scale structure formation process [49, 50].

In summary, previously it has been shown in literature that bosons of tiny mass with repulsive self-interaction forming a BEC at early times may account for DM content of the universe. Provided that the BEC, being trapped inside galaxies, can be considered as a system posed into a space-time with Schwarzschild metric, we consider the behavior of its macroscopic wave function characterized by oscillatory time dependence. We observed that for such a setup the strong energy condition is broken providing that vacuum-like equation of state at large galactocentric distances (in outer parts of galaxies). Thus, SFDM of ultra-light complex scalar field, which can form large size BEC around galaxies with observed properties of the dark matter, could also supply the dark energy component of the universe.

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